2

CHAPTER

ORBITAL MECHANISM AND LAUNCHING OF SATELLITE

INSIDE THIS CHAPTER

2.1. INTRODUCTION

The orbit of a satellite is geostationary and it has to be maintained geostationary at any cost. Mainly there are two problems regarding the satellite :

1. Launching and putting the satellite into geostationary orbit.
2. Maintaining the satellite into its orbit *i.e.* station keeping.

There are some standards recommended by **International Telecommunication Union (ITU)** and **International Consultive Committee (CCIR).** They prescribe the parking slots for individual countries and this helps in avoiding the unncessary losses or traffic capacity. The communication satellite is a very large investment and its life is limited. Therefore its full utilisation is very essential.

The communication satellites move around the earth as planets do around the sun and therefore the Kepler's laws are applicable to them also.

2.1.1. Kepler's First Law

It states that the path followed by a satellite around the primary will be an ellipse. An ellipse has two focal points shown as F_1 and F_2 in Fig. 2.1.

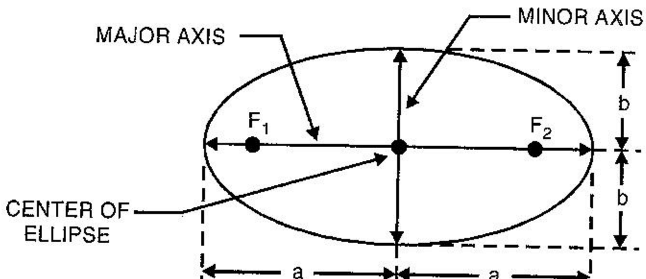

Fig. 2.1.

The center of the mass of the two body system, termed the **bary center**, is always centered on one of the foci. The semimajor axis is denoted by 'a' and semiminor axis is by 'b'. The eccentricity 'e' is given by

$$e = \frac{\sqrt{a^2 - b^2}}{a} \quad \text{...(2.1)}$$

The eccentricity and semimajor axis are two of the orbital parameters specified for satellite (spacecraft) orbiting the earth.

For $0 < e < 1 \rightarrow$ Elliptical orbit
$e = 0 \rightarrow$ Circular orbit

2.1.2. Kepler's Second Law

It states that for equal time intervals, a satellite will sweep out equal areas in its orbital plane, focussed at the body center as shown in Fig. 2.2.

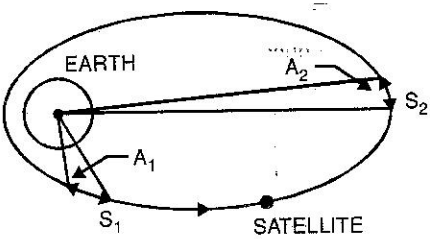

Fig. 2.2.

Assuming that the satellite travels distances S_1 and S_2 meters in 1 sec, then the areas A_1 and A_2 will be equal. The average velocity in each case is S_1 and S_2 meters per sec. and because of the equal area law, it follows that the velocity at S_2 is less than at S_1.

2.1.3. Kepler's Third Law

It states that the square of the periodic time of orbit is proportional to the cube of the mean distance between the two bodies. The mean distance is equal to semi-major axis, a

So, $$a^3 = \frac{\mu}{n^2} \quad \text{...(2.2)}$$

where n = Mean motion of the satellite in radians/sec.
μ = Earth's geocentric gravitational constant
$= 3.98 \times 10^{14}$ m^3/sec^2

with n in radians per second. The orbital period in sec is given by

$$T = \frac{2\pi}{n}$$

So $$\boxed{T = 2\pi\sqrt{\frac{a^3}{\mu}}} \quad \text{...(2.3)}$$

The Newton's gravitational law of forces also control the satellite orbits.

Some important points useful in the determination of orbit of satellites are :

- Distance between the satellite and earth station.
- Converge angle.
- Earth station pointing angle.
- Eclipse and solar interferences.

- Polarisation angle.
- Converge angle.
- Slant range.

Because of a continuous drift in the satellite orbit both in longitude and lattitude due to various disturbances, perfect stationary satellite orbit is not possible. So a satellite is constrained to remain within a window. Whose limits are defined by angular shift as seen from the center of the earth around the required nominal position, usually this window is 75 Km on the sphere containing the geostationary satellite orbit.

2.2. EQUATION OF THE ORBIT

Let us consider the rectangular co-ordinate system. The origin is at the centre of earth, and the z-axis extends through the north geographic pole. The co-ordinate system is right handed, it is fixed in space and the earth rotates on the z-axis. It is assume that the center of mass of the earth satellite system coincides with the center of mass of the earth at the origin.

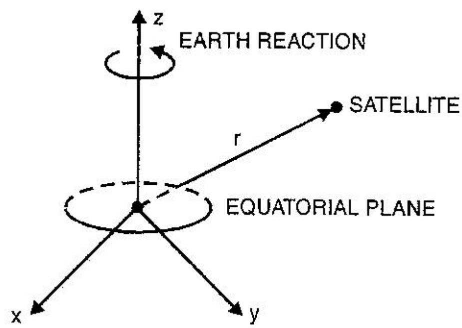

Fig. 2.3. Co-ordinate system used to describe the earth and a satellite.

The vector r locates the moving satellite with respect to the centre of the earth. If the mass of a satellite is m. The gravitational force F on the satellite is given by

$$F = -\frac{GMm\,\hat{r}}{r^2} \qquad \text{...(2.4)}$$

where, m = Mass of earth

$G = 6.672 \times 10^{-11}$ Nm/kg^2 (Universal gravitational constant)

$\hat{r}$ = Unit vector in the r direction

Product term = $GM = 3.98 \times 10^5$ km^3/s^2 (Kepler's constant) 'μ'

According to the Newton's second law

$$F = m\frac{d^2r}{dt^2}.\hat{r} \qquad \text{...(2.5)}$$

Equating the inertial force on satellite to the gravitational force, we have

$$-\frac{\mu m\hat{r}}{r^2} = m.\frac{d^2r}{dt^2}.\hat{r}$$

$$-\frac{\mu\hat{r}}{r^2} = \frac{d^2r}{dt^2}.\hat{r} \qquad \text{...(2.6)}$$

This is a **second order vector linear differential equation** and its solution will involve six undetermined constants called the **orbital elements.** Writing it as

$$r = r.\hat{r} \qquad ...(2.7)$$

and rearranging eqn. (2.6), we have

$$\frac{1}{r}.\frac{d^2r}{dt^2} + \frac{\mu r}{r^3} = 0 \qquad ...(2.8)$$

Motion defined by above equation is confined to a plane. Taking $r \times$ each term yields

$$r \times \frac{d^2r}{dt^2} = 0 \qquad ...(2.9)$$

Invoking the rule for finding the derivative of a product, it follows that

$$\frac{d}{dt}\left(r \times \frac{dr}{dt}\right) = \frac{dr}{dt} \times \frac{dr}{dt} + r \times \frac{d^2r}{dt^2} \qquad ...(2.10)$$

The cross product of any vector with itself is zero; hence equation (2.10) may be rewritten as

$$\frac{d}{dt}\left[r \times \frac{dr}{dt}\right] = 0 \qquad ...(2.11)$$

This is equivalent to

$$r \times \frac{dr}{dt} = h \qquad ...(2.12)$$

where, h = Constant

It is the **orbital angular momentum** of the satellite.

h can be a constant only if the orbit lies in a plane. So the problem of motion of satellite in three dimensions reduces to the problem of motion in a plane.

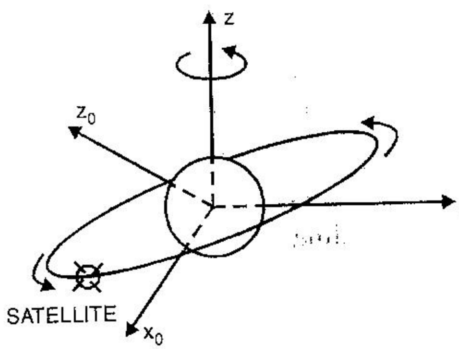

Fig. 2.4. Orbital plane co-ordinate system.

X_0 and Y_0 axis lie in the orbital plane and the z-axis is perpendicular to it. It is difficult to solve the eqn. (2.8) because of second derivative of r but it can be solved by expressing the equation in a rectangular co-ordinate system where the unit vectors are constant (Fig. 2.4). The first two co-ordinates lie in the plane and z_0 is normal to it. Expressing the eqn. (2.8) in terms of these co-ordinates yields

$$\hat{X}_0\left(\frac{d^2x_0}{dt^2}\right) + \hat{Y}_0\left(\frac{d^2y_0}{dt^2}\right) + \frac{\mu(x_0\hat{x}_0 + y_0\hat{y}_0)}{(x_0^2 + y_0^2)} = 0 \qquad ...(2.13)$$

Above equation can be solved easily if it is expressed in polar co-ordinate system (r_0, ϕ_0).

Using the transformations, we have

$$x_0 = r_0 \cos \phi_0 \quad ...(2.14)$$

$$y_0 = r_0 \sin \phi_0 \quad ...(2.15)$$

$$\hat{X}_0 = \hat{r}_0 \cos \phi_0 - \hat{\phi}_0 \sin \phi_0 \quad ...(2.16)$$

$$\hat{Y}_0 = \hat{\phi}_0 \cos \phi_0 - \hat{r}_0 \sin \phi_0 \quad ...(2.17)$$

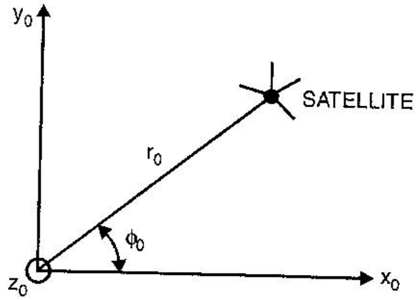

Fig. 2.5. Polar co-ordinates in the orbital plane.

and equating the $\hat{r}_0$ components of eqn. (2.13), we have

$$\frac{d^2 r_0}{dt^2} - r_0 \left(\frac{d\phi_0}{dt}\right)^2 = -\frac{\mu}{r_0} \quad ...(2.18)$$

Similarly equating $\hat{\phi}_0$ components, we have

$$r_0\left(\frac{d^2\phi_0}{dt^2}\right) + 2.\left(\frac{dr_0}{dt}\right)\left(\frac{d\phi_0}{dt}\right) = 0 \quad ...(2.19)$$

We may rewrite the eqn. (2.19) as

$$\frac{1}{r_0} \cdot \frac{d}{dt}\left(r_0^2 \cdot \frac{d\phi_0}{dt}\right) = 0 \quad ...(2.20)$$

and this is equivalent to

$$r_0^2 \cdot \frac{d\phi_0}{dt} = \text{a constant, } |\, h \,| = h \quad ...(2.21)$$

where h is the magnitude of the angular momentum vector in eqn. (2.12), writing it as

$$r_0 \left(\frac{d\phi_0}{dt}\right)^2 = \frac{h^3}{r_0^3} \quad ...(2.22)$$

Now substituting this expression into eqn. (2.18), we have

$$\frac{d^2 r_0}{dt^2} - \frac{h^2}{r_0^3} = -\frac{\mu}{r_0} \quad ...(2.23)$$

To find out the equation relating r_0 and ϕ_0, we have to eliminate t from eqn. (2.23). So we define a new variable 'u' by

$$u = \frac{1}{r_0} \quad ...(2.24)$$

So that, $$\frac{dr_0}{d\phi_0} = -\frac{1}{u^2} \cdot \frac{du}{d\phi_0} \quad ...(2.25)$$

and using the relationship, we have

$$\frac{dr_0}{dt} = \left(\frac{dr_0}{d\phi_0}\right)\left(\frac{d\phi_0}{dt}\right) - \left(\frac{dr_0}{d\phi_0}\right)\left(\frac{h}{r_0^2}\right)$$

$$= -h.\left(\frac{du}{d\phi_0}\right) \quad ...(2.26)$$

Now transforming $\left(\dfrac{d^2 r_0}{dt^2}\right)$ in eqn. (2.23) to

$$\frac{d^2 r_0}{dt^2} = -h^2 u^2 \left(\frac{d^2 u}{d\phi_0^2}\right) \qquad ...(2.27)$$

We may rewrite eqn. (2.23) as

$$\frac{d^2 u}{d\phi_0} + u = \frac{\mu}{h^2} \qquad ...(2.28)$$

According to the standards, the solution of this differential equation is

$$u = \frac{u}{h^2} + e \cos(\phi_0 - \theta_0) \qquad ...(2.29)$$

where c and θ_0 are constants and it can be determined by the boundary conditions. Representing the above equation in terms of r_0, we have

$$r_0 = \frac{1}{\dfrac{\mu}{h^2} + e \cos(\phi_0 - \theta_0)}$$

$$= \frac{\left(\dfrac{h^2}{\mu}\right)}{1 + \left(\dfrac{h^2}{\mu}\right) e \cos(\phi_0 - \theta_0)}$$

$$r_0 = \frac{P}{1 + e \cos(\phi_0 - \theta_0)} \qquad ...(2.30)$$

For $e < 1$, this is the equation of an ellipse. Whose semilatus spectrum P is given by

$$P = \frac{h^2}{\mu} \qquad ...(2.31)$$

where, Eccentricity $(e) = \dfrac{h^2 c}{\mu}$

for $e = 0$, orbit is a circle with the earth at its center.

The quantity θ_0 serves to orient the ellipse with respect to orbital plane axes x_0 and y_0 if θ_0 = zero, accordingly we can choose x_0 and y_0.

Then the equation of orbit will be

$$\boxed{r_0 = \frac{P}{1 + e \cos \phi_0}} \qquad ...(2.32)$$

Some of the **orbital parameters** are shown in Fig. 2.6.

The value of eccentricity e determines the type of orbit.

$$e = \begin{cases} 0 & \rightarrow \text{Circle} \\ <1 & \rightarrow \text{Ellipse} \\ 1 & \rightarrow \text{Parabola} \\ >1 & \rightarrow \text{Hyperbola} \end{cases}$$

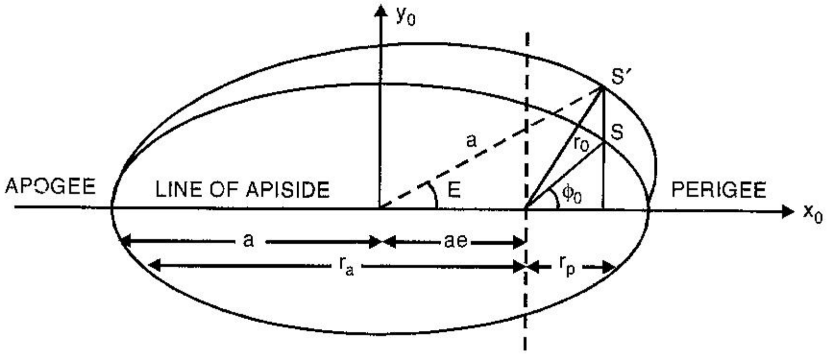

Fig. 2.6. Geometry of an elliptical orbit of communication satellite.

In the Fig. 2.6, a is the semi-major axis and b is semi-minor axis.

- Orbital period, $T = 2\pi\sqrt{\frac{a^3}{\mu}}$ ...(2.33)

$$\mu = GM = 3.98 \times 10^{14} \text{ m}^3/\text{s}^2$$

- Satellite velocity for a circular orbit $= \sqrt{\frac{\mu}{a}}$ ...(2.34)
- Satellite velocity at an orbital point S (distance to the centre of earth r) is

$$v_s^2 = \frac{2\mu}{r} - \frac{\mu}{a} \quad \text{...(2.35)}$$

$$x = r_0 \cos \phi_0 = a (\cos E - e) \quad \text{...(2.36)}$$

$$y = r_0 \sin \phi_0 = a \sin E (1 - e^2)^{1/2} \quad \text{...(2.37)}$$

- Eccentricity, $e = \frac{c}{a} = \frac{r_a - r_p}{r_a + r_p}$ ...(2.38)
- Semi-major axis, $a = \frac{(r_a + r_p)}{2} = \frac{p}{(1-e^2)}$ ...(2.39)
- Apogee distance, $r_a = a + c = a(1 + e)$ ...(2.40)
- Perigee distance, $r_p = a - c = a(1 - e)$ ...(2.41)
- Locus parameter, $P = a(1 - e^2) = \frac{2r_a r_p}{(r_a + r_p)}$ ...(2.42)
- Semi-minor axis, $b = a(1-e^2)^{1/2} = (r_a r_p)^{1/2}$ ...(2.43)

2.3. LOCATING THE SATELLITE IN THE ORBIT

The equation of orbit is,

$$r_0 = \frac{P}{1 + e \cos \phi_0}$$

and the semi-major axis is given as

$$a = \frac{P}{(1-e^2)}$$

So the equation of orbit may be rewritten as

$$r_0 = \frac{a(1-e^2)}{1+e\cos\phi_0} \qquad ...(2.44)$$

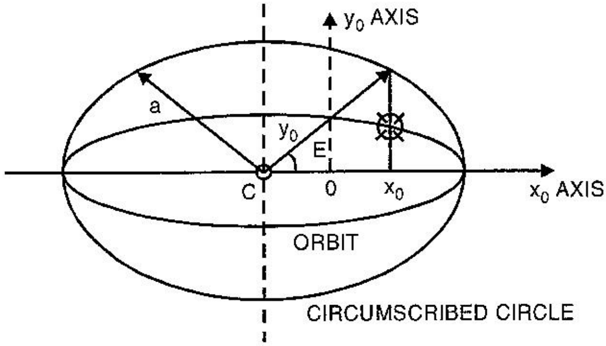

Fig. 2.7. Satellite position is specified in orbital plane co-ordinate system b > (x_0, y_0).

The angle ϕ_0 is measured from x_0 axis and is called the **true enomaly.** Positive x_0 axis passes through its perigee, ϕ_0 measures the angle from the perigee to the instantaneous position of the satellite. The rectangular co-ordinates of the satellite are given by

$$x_0 = r_0 \cos\phi_0 \qquad ...(2.45)$$

$$y_0 = r_0 \sin\phi_0 \qquad ...(2.46)$$

The orbital period T is the time required for the satellite to complete one revolution and travel 2π radian. The average angular velocity is

$$\eta = \frac{2\pi}{T} = \frac{\mu^{1/2}}{a^{3/2}} = \frac{1}{a}\left(\frac{\mu}{a}\right)^{1/2} \qquad ...(2.47)$$

If the time required for a spacecraft moving at this angular velocity to go around any circle is T sec., then the object will go around the circumscribed circle with a constant angular velocity η and it will complete one revolution as the satellite requires to complete one orbital revolution.

To find the equation of the orbit, we have to eliminate 't' from the equation of motion. But for the spacecraft location 't' is required.

We can relate the linear velocity v to r_0 and ϕ_0 by

$$v^2 = \left(\frac{dx_0}{dt}\right)^2 + \left(\frac{dy_0}{dt}\right)^2 = \left(\frac{dr_0}{dt}\right)^2 + r_0^2\left(\frac{d\phi_0}{dt}\right)^2 \qquad ...(2.48)$$

It can be shown that

$$v^2 = \frac{\mu}{a}\left(\frac{2a}{r_0} - 1\right) \qquad ...(2.49)$$

From equations (2.21), (2.22) and (2.39)

$$r_0^2\left(\frac{d\phi_0}{dt}\right)^2 = \frac{h^2}{r_0^2} = \frac{\mu p}{r_0^2} = \frac{\mu a(1-e^2)}{r_0^2} \qquad ...(2.50)$$

then eqn. (2.48) will be

$$\frac{\mu}{a}\left(\frac{2a}{r_0}-1\right) = \left(\frac{dr_0}{dt}\right)^2 + \frac{\mu a}{r_0}(1-e^2) \qquad ...(2.51)$$

and

$$\frac{dr_0}{dt} = \left\{\frac{\mu}{ar_0^2}\left[a^2e^2-(a-r_0)^2\right]\right\}^{1/2} \qquad ...(2.52)$$

Solving the above equation for dt and multiplying by the mean angular velocity 'η', we have

$$\eta . dt = \frac{r_0}{a}\frac{dr_0}{[a^2e^2-(a-r_0)^2]^{1/2}} \qquad ...(2.53)$$

Let us consider the geometry of the circumscribed circle as shown in Fig. 2.7. A line from the center of ellipse to the point A makes an angle E with the x_0 axis; E is called **eccentric anomaly** of the satellite and it is related to radius r_0 by

$$r_0 = a(1-e\cos E) \qquad ...(2.54)$$

$$a-r_0 = ae\cos E \qquad ...(2.55)$$

Expressing the eqn. (2.53) in terms of E

$$\eta . dt = (1-e\cos E)\, dE \qquad ...(2.56)$$

if t_p = Time of perigee

It is the time of closest approach to the earth when the satellite is crossing x_0 axis and time when E is zero. Integrating both sides of eqn. (2.57), we have

$$\eta(t-t_p) = E - e\sin E \qquad ...(2.57)$$

Left side of eqn. (2.57) is called **mean anomaly,** m

$$M = \mu(t-t_p) = E - e\sin E \qquad ...(2.58)$$

where, M = Arc length in radian

η = Mean angular velocity

2.4. LOCATING THE SATELLITE WITH RESPECT TO EARTH

To locate the satellite at a point (x_0, y_0, z_0) in the recangular co-ordinate system of the orbital plane, we have to develop transformations that permits the satellite to be located from a point to the rotating surface of the earth.

We start with a fixed rectangular co-ordinate system (x_i, y_i, z_i) called the **geocentric co-ordinate system** whose origin is at the center of earth. It is shown in Fig. 2.8. z_i axis coincides with the earth's axis of rotation. X_i points towards a fixed location in space and is called the **first point of aries.** This is the direction of a line from the center of earth through the center of sun at the vertical equinox, at the instant when the subsolar point

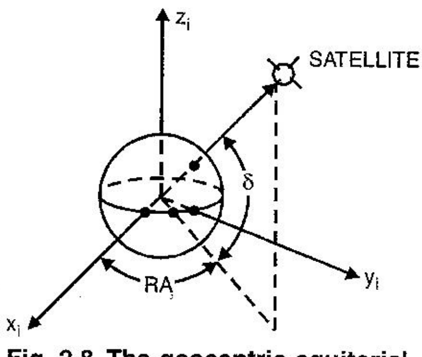

Fig. 2.8. The geocentric equitorial system.

crosses the equator south to north. This co-ordinate system translates, as the earth revolves around the sun and it moves through space. X_i direction is the same as the earth's position. The plane (x_i, y_i) contains the earth's equator and is called **equitorial plane.**

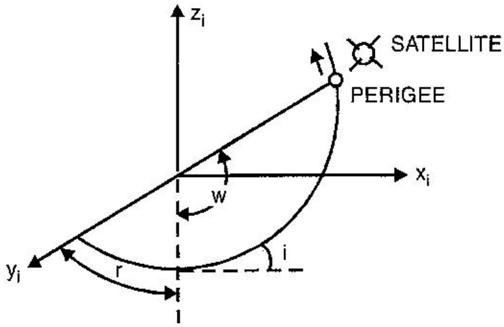

Fig. 2.9. Locating the orbit in geocentric equitorial system (Satellite penetrates the equitorial plane).

The measurement of angular distance eastward in equitorial plane from the x_i axis is called **right ascension** and is denoted by symbol R_A. The points at which the orbit penetrates the equitorial plane are called **nodes.** Satellite moves upward through the equitorial plane at the **ascending node** and downward at any **descending node**. The right ascersion of ascending node is denoted by 'r'. The angle that the orbital plane makes with the equitorial plane is called **inclination** 'i' and all these quantities are shown in Fig. 2.9.

To locate the orbital co-ordinate system with respect to equitorial co-ordinate system, there is a need of w **argument of perigee.** This is the angle measured along the orbit from the ascending node to the perigee.

The orbital plane co-ordinate (x_0, y_0, z_0) of a satellite is related to the satellite co-ordinates (x_i, y_i, z_i) by a linear transformation and is given by

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \begin{bmatrix} (\cos r)(\cos w) - (\sin r)(\sin w)(\cos i) & -(\cos r)(\sin w) - (\sin r)(\cos i)(\cos w) & (\sin r)(\sin i) \\ (\sin w).(\cos w) + (\cos r)(\cos i).(\sin w) & -(\sin w)(\sin r) + (\sin r)(\cos i)(\cos w) & -(\cos w)(\sin i) \\ (\sin i)(\sin w) & (\sin i)(\cos w) & (\cos i) \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \quad ...(2.59)$$

One more co-ordinate transformation is required to locate the satellite w.r.t. a point on the rotating earth. So we need a rotating rectangular system (x_r, y_r, z_r) attached to the earth whose $x - y$ plane and z-axis corresponds to the geocentric equatorial system, as shown in Fig. 2.10. The rotating system turns at angular velocity Ω_e and T_e measured the elapsed time since x_r coincides with x_i axis.

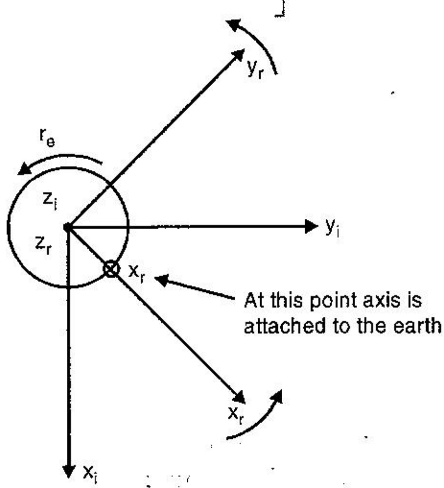

Fig. 2.10. The relationship between the rotating co-ordinate system (x_r, y_r, z_r) and the geocentric equatorial system (x_i, y_i, z_i).

The co-ordinates of satellite in the rotating system are related to the co-ordinates in the geocentric equitorial system and given by

$$\begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = \begin{bmatrix} \cos(r_e T_e) & \sin(r_e T_e) & 0 \\ -\sin(r_e T_e) & \cos(r_e T_e) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \quad ...(2.60)$$

The value of $(\Omega_e T_e)$ at any time t (in min) can be expressed as

$$\Omega_e T_e = \alpha_{(q,0)} + 0.25068447t \text{ degree} \quad ...(2.61)$$

where, $\alpha_{(q,0)} = 99.6909833 + 36000.7689T_c + 0.00038708\,T_c^2$ degree ...(2.62)

(right ascersion)

and T_c is the elapsed time in Julian centuries between $0h$ UT and it is calculated by

$$T_c = \frac{(JD - 2415020)}{36525} \text{ Julian centuries} \quad ...(2.63)$$

where, JD = Julian day

2.5. ORBITAL ELEMENTS

To specify the co-ordinates of the satellite at any time, six quantities are required. These are :

1. Eccentricity (e)
2. Semi-major axis (a)
3. Time of perigee (t_p)
4. Right ascension of ascending mode (r)
5. Inclination (i)
6. Argument of perigee (w)

Sometimes mean anomaly (M) at a given time can be substituted for t_p.

2.6. APOGEE AND PERIGEE HEIGHTS

Apogee is that point in the satellite orbit that is the farthest from the center of earth and the apogee distance is shown by r_a.

$$r_a = a(1 + e) \quad ...(2.64)$$

The satellite velocity is the lowest at the apogee point.

Perigee is the point in the orbit that is nearest to the earth. The perigee distance is shown by 'r_p' and can be written as,

$$r_p = a(1 - e) \quad ...(2.65)$$

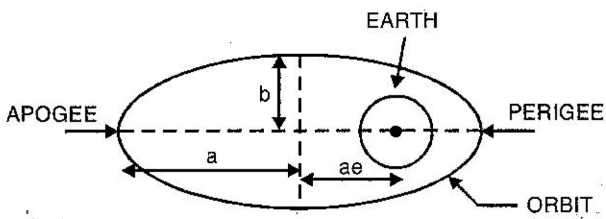

Fig. 2.11.

In order to find out the apogee and perigee heights. The radius of the earth must be subtracted from the radi lengths.

2.7. ORBIT PERTURBATIONS

The type of orbit described for satellite, referred to as **Keplerian orbit.** It is elliptical for the special case of an artificial satellite orbiting the earth. In practice, the satellite and the earth respond to many other influences like :

- assymetry or the earth's gravitational field.
- The gravitational field of the sun and moon.
- Solar radiation pressures.
- Some other factors like atmospheric drag.

But it does affect low orbiting earth satellites below about 1000 Km.

2.7.1. Effect of a Non-spherical Earth

The earth is not perfectly spherical, there is an equatorial bulge and a flattering at the poles and this type of shape is known as **oblate spheroid.**

Kepler's third law gives the nominal mean motion as,

$$n_0 = \sqrt{\frac{\mu}{a^3}} \qquad ...(2.66)$$

When the earth oblateness is taken into account, the mean motion n is modified to,

$$n = n_0\left[\frac{1+K_1(1-1.5\sin^2 i)}{a^2(1-e^2)^{1.5}}\right] \qquad ...(2.67)$$

where, K_1 is a constant which evaluates to 66063.1704 km^2.

The orbital period of the earth's oblateness is termed the **anomalistic period.**

The anomalistic period is

$$P_A = \frac{2\pi}{n} \text{ sec.} \qquad ...(2.68)$$

where, n = radians per second

Equation (2.67) may be solved for 'a' by finding the **root** of the following equation.

$$n - \sqrt{\frac{\mu}{a^3}}\left[1+\frac{K_1(1-1.5\sin^2 i)}{a^2(1-e^2)^{1.5}}\right] = 0 \qquad ...(2.69)$$

The oblateness of the earth also produces two relations of the orbital plane. The first is known as **regression of the nodes**, is where the nodes appear to slide along the equator. Line of nodes, which is in the equatorial plane, rotates about the center of earth. Thus r, the right ascension of the ascending node shifts its position. The second effect is **rotation of apsides** in the orbital plane.

In addition to the equitorial bulge, the earth is not perfectly circular in the equitorial plane, it has a small eccentricity of the order of 10^{-5}. This is referred to as the **equatorial ellipticity.** This effect setup a gravity gradient which has some effects on satellites in geostationary orbit. The gravity gradient causes the satellite in geostationary orbit to drift to one or two stable points. This effect of equatorial ellipticity is negligible on most other satellite orbits.

2.7.2. Effects of the Sun and Moon

Because of the gravitational attraction of sun and moon there is a change in orbital inclination of a geosynchronous satellite that change with time. These forces would increase the orbital inclination from an initial 0° at launch to 14.67°, 26.6 years later. But no commercial satellite has a 26 years useful life time. The rate of change varies with the inclination of the moon's orbit, but values of about 0.86° per year are quoted for the 1970-80 time period.

2.8. LOOK ANGLE'S

The co-ordinates to which a earth station antenna must be pointed to communicate with a satellite are called the **look angles**. These angles are :

- Azimuth angle (A)
- Elevation angle (E)

These angles can be calculated on the basis of the knowledge latitude (ϕ) and relative longitude (θ) of the earth station.

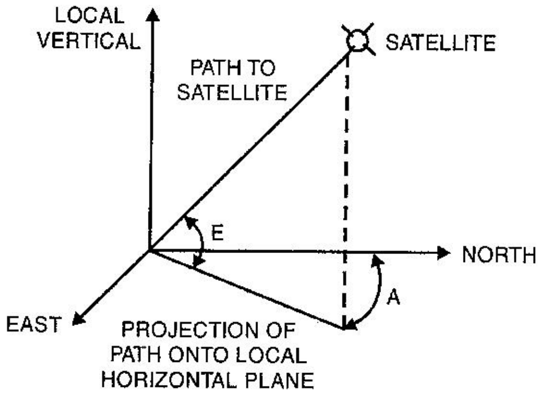

Fig. 2.12. Measurement of azimuth and elevation angle.

2.8.1. Azimuth Angle

It is defined as the angle by which the antenna pointing at the horizon must be rotated clockwise around its vertical axis from the geographical north to bring the antenna boresight into the vertical plane containing the satellite direction. Its value varies in between 0° and 360°.

True azimuth angle can be calculated as

S. No.	True Azimuth	Earth station quadrant	
1.	$A = 180° - \gamma$	ES west to satellite	ES in northern region
2.	$A = 180° + \gamma$	ES east of satellite	
3.	$A = \gamma$	ES west of satellite	ES in southern region
4.	$A = 360° - \gamma$	ES east of satellite	

and the angle γ can be calculated as

$$\boxed{\gamma = \text{arc}\, tg\left(\frac{tg\,.\,\theta}{\sin\phi}\right)} \qquad ...(2.70)$$

where, θ = Relative earth station longitude (in degree)

ϕ = Earth station lattitude (in degree)

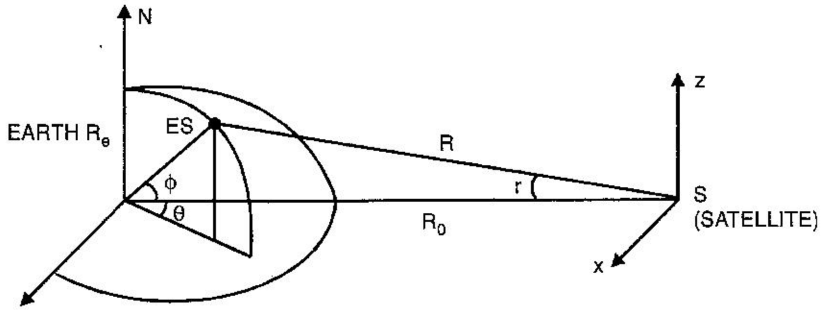

Fig. 2.13. Calculation of angle *r* from the geometry of satellite orbit in the reference system.

2.8.2. Elevation Angle

It is defined as the angle by which the antenna boresight should be rotated in the vertical plane that contains the satellite direction from the horizontal to the satellite direction.

It is also calculated from the given formula.

$$\boxed{E = \frac{\text{Arc}\, tg\left(\cos\theta\,.\cos\phi - \dfrac{R_e}{R_e + R_0}\right)}{\left[1-(\cos\theta\,.\cos\phi)^2\right]^{1/2}}} \qquad ...(2.71)$$

where, R_e = Earth radius = 6378 Km

R_0 = Satellite distance = 35786 Km

Polarization (ψ) angle is defined as the angle between the polarization plane of a linear polarized wave transmitted by the satellite and the polarization plane of earth station antenna, it may be calculated as

$$\tan\psi = \frac{\sin\theta}{\tan\phi} \qquad ...(2.72)$$

2.9. EARTH COVERAGE AND SLANT RANGE

From the Fig. 2.14, the co-ordinates of an earth station may be written as

$$x = R_0 \cos\phi\,.\,\sin\theta$$
$$y = R_0 + R_e(1-\cos\phi\,.\,\cos\theta)$$
$$z = R_e \sin\phi$$

The **slant range** R from the earth station to the satellite can be given as

$$R^2 = R_0^2 + 2R_e(R_0 + R_e)(1 - \cos\phi \cdot \cos\theta) \qquad ...(2.73)$$

for $\frac{R_e}{R_0} \simeq 0.178$, it will be

$$\left(\frac{R}{R_0}\right)^2 = 1 + 0.42\,(1 - \cos\theta \cdot \cos\phi) \qquad ...(2.74)$$

From the above equation, it is clear that the maximum value of $\left(\frac{R}{R_0}\right)^2$ is 1.356 and if R^2 is approximately equal to R_0^2 then there will be a error of 1.3 dB in power link. The earth can be covered by a single beam communication satellite and it is desired that it should cover the entire country and this converge should be as maximum as possible. The maximum coverage of the earth by a geosationary satellite is shown in Fig. 2.14.

$$\theta_s = 180° - (90° + E + \alpha) = 90° - E - \alpha \qquad ...(2.75)$$

The maximum geometric coverage is given by the portion of the earth within a cone with the satellite at its apex and tangent to the earth's surface. The coverage angle or the apex angle of this cone is $2\alpha_{max.}$ or

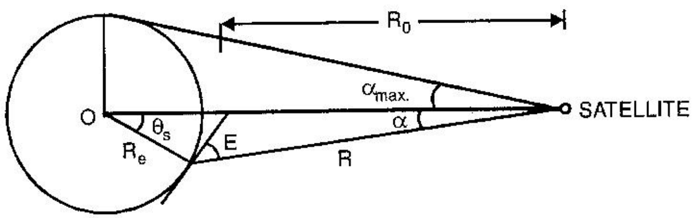

Fig. 2.14. Elevation angle and coverage angle with reference to synchronous satellite.

$$2\,arc\sin\left(\frac{R_e}{R_e + R_0}\right) = 17.4° \qquad ...(2.76)$$

θ_s represents the angular radius of the satellite footprint and is called **central angle.**

For a geostationary orbit θ_s can be calculated by putting $\alpha = \alpha_{max}$ and $E = 0$ in eqn. (2.71)

So, $\theta_s = 81.3°$

But for all practical purpose an earth station requires a minimum elevation angle of 5° above the horizon.

Then $\theta_s = 76.3°$

So not more than 76.3° of northern or southern latitudes will be covered by the geosynchronous satellite.

Slant range in terms of elevation angle may given as

$$R^2 = (R_e + R_0)^2 + R_e^2 - 2R_e(R_e + R_0) \times \sin\left[E + \sin^{-1}\frac{R_e}{R_e + R_0}\cos E\right] \quad ...(2.77)$$

For $E_{min} = 5°$

Maximum slant range will be, $R_{max} = 41.127$ Km

it gives a satellite **round trip delay of,** $\frac{2d}{c} = 0.274$ s

where, c = Velocity of light

2.10. ECLIPSE EFFECTS

Earth and moon affect the working of a satellite because of solar eclipse. So the periodicity and duration of solar eclipse are important. Solar eclipse due to earth lasts for several days so it is more important than the eclipse due to moon on the communication satellite. Summer solstice occurs on 21st June and winter solstice is on 21st December. Similarly spring and autumn equinoxes occur on 21st March and 21st September respectively.

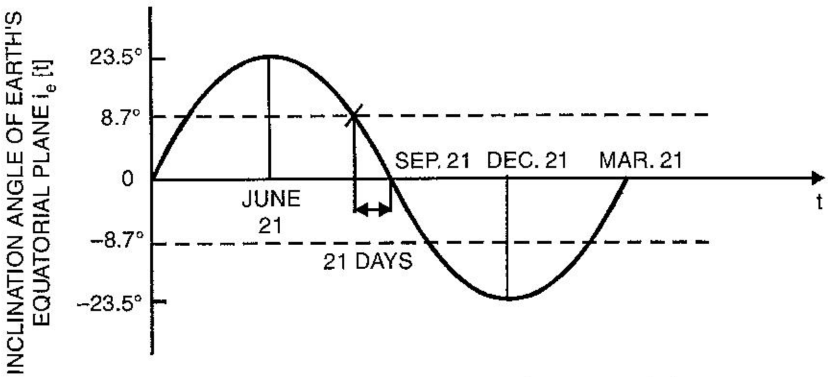

Fig. 2.15. Variation of the earth's equatorial plane inclination angle i_e (t).

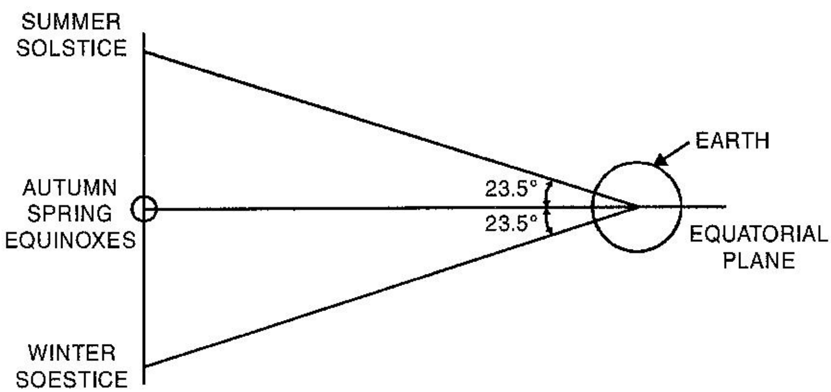

Fig. 2.16. Apparent movement of the sun relative to the earth.

In the Fig. 2.15, the inclination angle of the earth's equatorial plane i_e (t) with respect to the sun's direction is shown. Here the angle i_e (t) is in degree and annual variation is sinusoidal in nature. The apparent movement of the sun relative to the earth is shown in Fig. 2.16.

Mathematically the variation of $i_e(t)$ can be written as

$$i_e(t) = 23.5 \sin \frac{2\pi t}{T} \qquad ...(2.78)$$

where, T = Annual period equal to 365 days.

and $$i_e(t)]_{max} = 23.5° \qquad ...(2.79)$$

$i_e(t)$ is maximum on summer and winter solstices and it is zero at spring and autumn equinoxes.

The satellite is always illuminated at the solestice. it goes the maximum duration of eclipse during equinoxes.

The solar eclipse during equinox is shown in Fig. 2.17. The earth shadow is considered to be a cylinder of constant diameter and the maximum shadow angle at equinox is given as

$$\phi_{max} = 180° - 2\cos^{-1}\left(\frac{R_e}{R_e + R_0}\right) \qquad ...(2.80)$$

It is same as that of coverage angle. Substituting the value, R_e = 6378 Km, R_0 = 35786 Km (satellite attitude)

$$\phi_{max} = 17.4°$$

Duration of eclipse whose period is 24 hours is given as

$$t_{max.} = \frac{17.4°}{360°} \times 24 \cong 69.4 \text{ min.}$$

$$\simeq 1.16 \text{ hrs.}$$

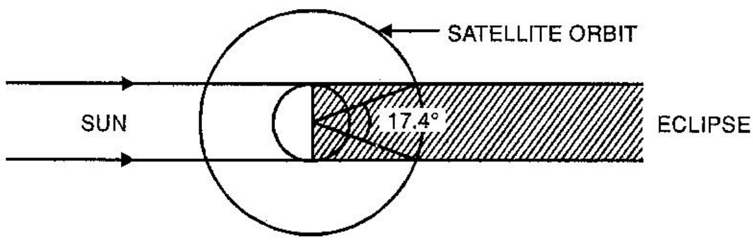

Fig. 2.17. Eclipse due to earth when sun is at equinox.

The relative position of a sun is calculated from the first ellipse day before an equinox and the last day of eclipse after an equinox. In this case, the inclination angle is given as

$$i_e = \frac{1}{2}\phi_{max} = 8.7° \qquad ...(2.81)$$

Corresponding to this inclination angle the time 't' from the first day of eclipse to the equinox and the time from the equinox to the last day of eclipse may be calculated as

$$t = \frac{365}{\pi}\sin^{-1}\left(\frac{87}{23.5}\right) \simeq 21 \text{ days} \qquad ...(2.82)$$

So to reach the equinox from the first day of eclipse 21 days are required. Thus in total for 42 days the solar eclipse near equinox affects the satellite working.

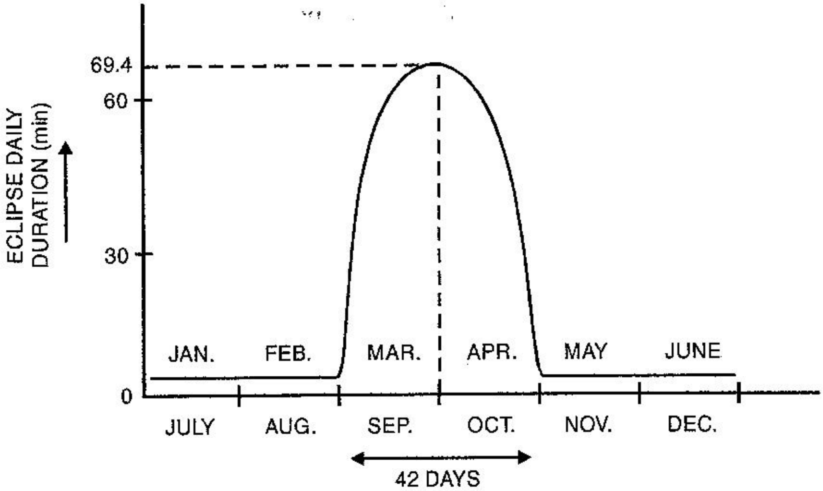

Fig. 2.18. Daily duration of solar eclipse caused by earth.

The solar eclipse caused by moon to the geostationary satellite occurs when the moon passes in front of the sun. It occurs irregulary in duration often it occurs twice within a period of 24 hours and it may range from a few minutes to over two hours period with an average duration of about 40 minutes. In comparison with earth solar eclipse, the number of moon solar eclipse range from zero to four with an average of two per year.

2.11. STATION KEEPING

Station keeping is the process of maintenance of satellite's attitude against different factors that can cause drift with time. Satellites need to adjust their orbit timely because natural forces induces a progressive drift even initially the satellite is placed in correct orbit. The factors that produce drift include :

- Minor gravitational perterbations due to sun and moon.
- Solar radiation pressure.
- The effect of non-spheric earth.

The orbital adjustments are normally carried out by releasing jets of gas or by firing small rockets tied around the body of satellite.

It is already mentioned that a window corresponding to a drift of 75 Km on the sphere containing the geostationary satellite orbit is around. It corresponds to an angular shift of ±0.1° in longitude for fixed satellite and also for broadcasting satellite services. This station keeping accuracy has been internationally standardized by CCIR. It is essential that in station keeping orbit correction techniques propellent consumption should be minimum. The strategy usually has certain steps, firstly the direction and the speed of drift of the satellite is determined using a new series of measurement. After that calculations the amplitude and direction of velocity increments are required to modify the orbital parameters.

Compensation of the inclination increments corresponds to North-South (N-S) station keeping and requires a thrust impulse perpendicular to the orbital plane. East-west (E-W) station keeping are obtained by applying thrusts in the orbital plane and it corresponds to ±0.1° E-W accuracy. If the life time of satellite is T_L years, maximum value of inclination angle will be

$$i_{max.} = \frac{1}{2} i_{av.} \cdot T_L \qquad ...(2.83)$$

where, i_{av} = Natural annual change of inclination averaged over the life time T_L.

2.12. GEOSTATIONARY AND OTHER ORBITS

Geostationary Orbit

A geostationary orbit is one in which a satellite moves in the orbit at exactly the same speed as the earth turns and, at the same lattitude specifically zero, the lattitude of the equator.

It is an equitorial circular orbit at such a height from earth's surface that a satellite moving from west to east has a velocity so as to orbit the earth once in a time equal to the time taken by earth to complete one rotation about its axis. This time is about 24 hours. In such cases the satellite appears stationary to an observer on earth. The orbital radius of geostationary satellite is 42164.2 Km. The movement of satellite is in synchronism with earth's rotation. All geostationary satellites are geosynchronous but vice versa is not true.

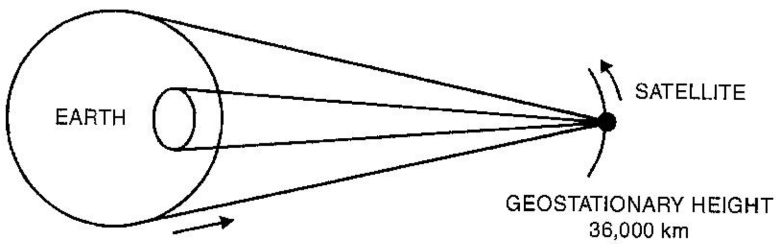

Fig. 2.19. Geostationary orbit periods of revolution equals the earth's rotational period.

The essential requirements for an orbit to be geostationary are :

(1) The satellite should have the angular velocity equal to that of the earth *i.e.* it should be geosynchronous.

(2) The orbit should have zero degree inclination with the equatorial plane. Any other inclination will not make the satellite staionary for some latitudes, even though it might be geosynchronous.

(3) For zero degree inclination, the orbit must be in the earth's equatorial plane.

(4) The satellite should travel eastward.

(5) As the earth's angular velocity is constant, the orbit has to be circular.

The above criteria can be fulfilled by one orbit only. Hence there is only one geostationary orbit. At present it is the most widely used orbit and is known as **'dark orbit'.**

Polar orbit : An orbit which passes over the poles. Polar orbits are relevant for those countries which are located near the poles and it is shown in Fig. 2.20.

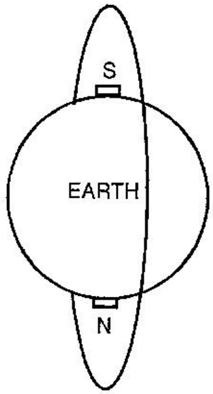

Fig. 2.20. Polar orbit.

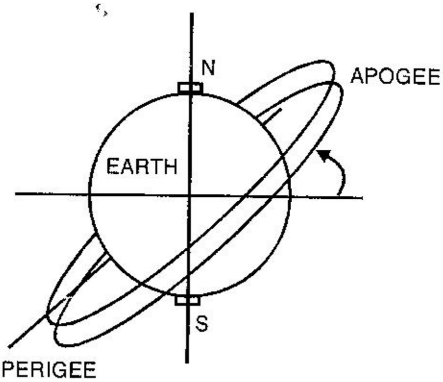

Fig. 2.21. Molniya orbit.

Molniya Orbit

It is a highly inclined, highly eccentric, elliptical orbit with an orbital period of about 12 hours. It is popular with russion communication satellites and it serves the purpose of a geosynchronous orbit for high latitude regions. Molniya orbits are capable of providing uninterrupted service if three satellites are placed at different phases and it is shown in Fig. 2.21.

Parking Orbit

Parking orbit is a temporary earth orbit during which the space vehicle is checked out and, its trajectory carefully measured to determine the amount and time of increase in velocity required to send it into a final orbit or into space in the desired direction. (Fig. 2.22)

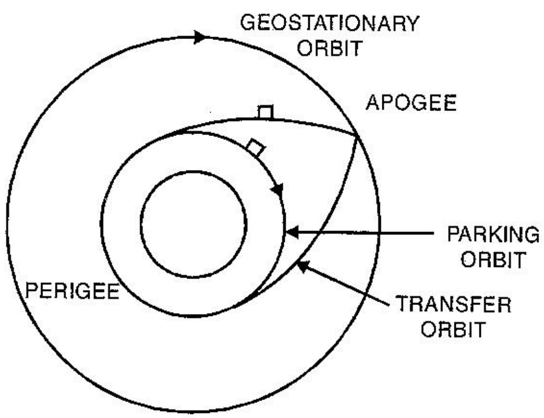

Fig. 2.22. Parking orbit.

Equatorial Orbit

It is a orbit in the plane of equator.

Inclined Orbit

Inclined orbits have an inclination between 0° (equatorial orbit) and 90° (polar orbit). These orbits may be determined either by the region on earth or by the lattitude on launch sites. These satellites are not sun-synchronous.

2.13. MECHANISM OF LAUNCHING A SATELLITE

Theoretically a satellite could be placed into geosynchronous orbit in one step but practically it is a two or three step process. The placement of satellite in the geostationary orbit is carried out on the principle of **Hohmann Transfer** and it is shown in Fig. 2.23.

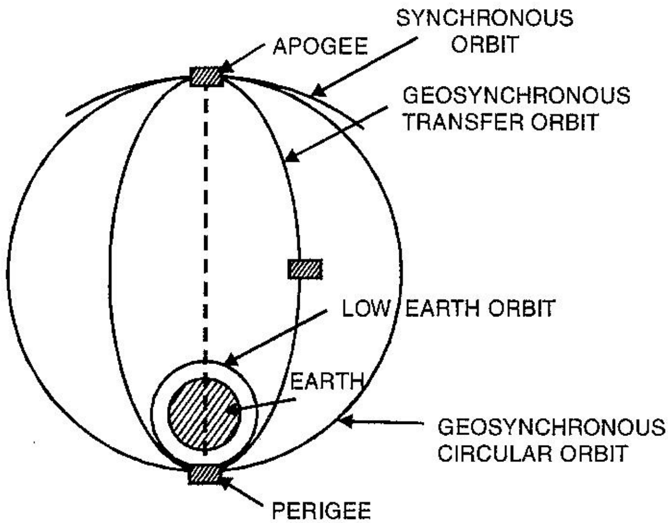

Fig. 2.23. Satellite placement in geostationary orbit.

There are **three techniques :**

(*i*) In the first technique, first the satellite is placed in lower circular earth orbit at an altitude of around 300 Km. Then a velocity increment is required and it is carried out by various auxiliary propulsion stages. One velocity increment changes the satellites lower circular earth orbit into an elliptical transfer orbit with a perigee of about 300 Km and apogee at 42164.2 Km (radius of geosynchronous orbit). Then a second velocity increment is required and it finally places the satellite into the desired orbit. **Space transport system (STS)** follows this technique.

(*ii*) The second technique has been used by **expandable launch vehicles** such as Ariane, Delta or Atlas-centaur launchers. In this case there is no initial circular orbit and the vehicle provides the necessary velocity at the perigee of the elliptical transfer orbit. Thus here only one velocity increment is required from the satellite at the apogee.

(*iii*) The third technique has been used by **special expendable launch vehicles** such as US Titan IIII C and the USSR Proton launchers. In this case satellites are directly placed into geostationary orbit.

The satellite velocity at the apogee and perigee can be calculated as given below :

$$v_s^2 = \frac{2\mu}{r} - \frac{\mu}{a} \qquad \text{...(2.84)}$$

where, μ = Gravitational constant

a = Semi-major axis

r = Distance from the center of earth to the point where velocity is required.

SOLVED QUESTIONS

Example 1. The orbital period of a satellite is 650 min. Determine the semi-major axis of the elliptical orbit.

Solution. Given that,

$$T = 650 \text{ min} = 650 \times 60 \text{ sec.} = 39000 \text{ sec.}$$

$$\mu = GM$$

$$= 6.67 \times 10^{-11} \times 5.98 \times 10^{24} = 39.8 \times 10^{13} \text{ Nm}^2\text{/kg}$$

The orbital time period (T) is given by

$$T = 2\pi\sqrt{\frac{a^3}{\mu}} = 2\pi\sqrt{\frac{a^3}{GM}}$$

$$39000 = 3.14 \times 2\sqrt{\frac{a^3}{39.8 \times 10^{13}}}$$

$$\Rightarrow \quad \sqrt{\frac{a^3}{39.8 \times 10^{13}}} = 6210.191$$

$$a^3 = 1534945636.7398 \times 10^{13}$$

$$a = 535.424 \times 10^4$$

$$\boxed{a = 5350 \text{ Km}} \quad \textbf{Ans.}$$

Example 2. The apogee and perigee of an elliptical satellite orbits are 3000 Km and 200 Km. Determine the eccentricity, semi-major axis and semi-minor axis.

Solution. Given that,

$$r_a = 3000 \text{ Km}$$

$$r_p = 200 \text{ Km}$$

Eccentricity, $$e = \frac{r_a - r_p}{r_a + r_p} = \frac{3000 - 200}{3000 + 200}$$

$$\Rightarrow \quad \boxed{e = 0.875} \quad \textbf{Ans.}$$

Semi-major axis, $$a = \frac{r_a + r_p}{2} = \frac{3000 + 200}{2}$$

$$\Rightarrow \quad \boxed{a = 1600 \text{ km}} \quad \textbf{Ans.}$$

Semi-minor axis, $b = \frac{r_a - r_p}{2} = \frac{3000 - 200}{2}$

$\Rightarrow$ $\boxed{b = 1400 \text{ km}}$ **Ans.**

Example 3. A geostatinary satellite moving in an equatorial circular orbit is at a height of 35786 Km from the earth's surface. If the radius of the earth is 6378 Km. Determine the theoretical maximum coverage angle and maximum slant range.

Solution. Given that,

$$R_0 = 35786 \text{ Km}$$
$$R_e = 6378 \text{ Km}$$

For theoretical maximum coverage angle,

Elevation angle, $E = 0$

$$\text{Maximum coverage angle} = 2\alpha_{max.}$$
$$= 2\sin^{-1}\left(\frac{R_e}{R_e + R_0}\cos E\right)$$
$$= 2\sin^{-1}\left(\frac{6378}{6378 + 35786}\cos 0\right)$$
$$= 17.4°$$

If the maximum slant range is R then

$$R^2 = R_e^2 + (R_e + R_0)^2 - 2R_e(R_e + R_0) \times \sin\left[E + \sin^{-1}\frac{R_e}{R_e + R_0}\cos E\right]$$
$$= (6378)^2 + (6378 + 35786)^2 - 2 \times 6378\,(6378 + 35786)\sin(8.7°)$$
$$= 40678884 + 1777802896 - 537843984 \times 0.1512$$
$$R^2 = 1737139041$$

$\Rightarrow$ $\boxed{R = 41679 \text{ Km}}$ **Ans.**

Example 4. A satellite is moving in a circular orbit at a height of 200 Km above the surface of earth. Determine its orbital velocity. ($G = 6.67 \times 10^{-11}$ Nm^2/kg^2, $m = 5.98 \times 10^{+24}$ kg)

Solution. The orbital velocity (V) is given by

$$V = \sqrt{\frac{\mu}{(R + H)}}$$

where,
$$\mu = GM$$
$$= 6.67 \times 10^{-11} \times 5.98 \times 10^{24}$$
$$= 39.8 \times 10^{13} \text{ Nm}^2/\text{kg}$$
$$R = \text{Radius of earth} = 6370 \text{ Km}$$
$$H = 200 \text{ Km}$$

Putting all the values, we get

$$V = \sqrt{\frac{39.8 \times 10^3}{(6370 + 200) \times 10^3}}$$

$$= 10^5 \sqrt{\frac{39.8}{6570}} \Rightarrow 10^4 \sqrt{0.6057}$$

$$= 10^4 \times 0.778$$

$$\boxed{V = 7.78 \text{ km/sec.}} \quad \textbf{Ans.}$$

Example 5. Determine the escape velocity for an object which is launched from the surface of earth to a point where the radius of earth is 6360 Km. ($G = 6.67 \times 10^{-11}$ Nm^2/kg^2, $M = 5.98 \times 10^{24}$ kg).

Solution. Escape velocity is given by

$$V_{escape} = \sqrt{\frac{2\mu}{r}}$$

where, $\mu = GM = 39.8 \times 10^{13}$ Nm^2/kg

$r = 6360$ Km

Putting these values, we get

$$V_{escape} = \sqrt{\frac{2 \times 39.8 \times 10^{13}}{6360 \times 10^3}} = \sqrt{\frac{79.6}{6360}} \times 10^5 = 11.2 \text{ Km/sec.}$$

$$\boxed{V_{escape} = 11.2 \text{ Km/sec.}} \quad \textbf{Ans.}$$

Example 6. A satellite is moving in a circular orbit at a height of 150 Km above the surface of earth. If the radius of earth is 6360 Km, determine the orbital velocity and orbital period of the satellite. ($G = 6.67 \times 10^{-11}$ Nm^2/kg, $M = 5.98 \times 10^{24}$ kg)

Solution. Given that, $H = 150$ Km, $R = 6360$ Km

$$\mu = GM = 6.67 \times 10^{-11} \times 5.98 \times 10^{24} = \mathbf{39.8 \times 10^{13}}$$

Orbital velocity of a satellite is given by

$$V_{orbital} = \sqrt{\frac{\mu}{(R + H)}} = \sqrt{\frac{GM}{(R + H)}}$$

Putting all values we get,

$$= \sqrt{\frac{39.8 \times 10^{13}}{(6360 + 150) \times 10^3}}$$

$$= \sqrt{\frac{39.8 \times 10^{13}}{6510 \times 10^3}} = 10^4 \times \sqrt{\frac{3980}{6510}}$$

$$\Rightarrow \quad 10^4 \times \sqrt{0.611367} = 10^4 \times 0.782$$

$$\boxed{V_{orbital} = 7.82 \text{ Km/sec.}} \quad \textbf{Ans.}$$

Orbital period of a satellites

$$= \frac{2\pi(R+H)}{V_{orbital}}$$

$$= \frac{2 \times 3.14 \times (6360+150)}{7.82} = \frac{6.28 \times 6510}{7.82}$$

$$= 5227.98 \text{ sec.}$$

$$\boxed{T_{\text{orbital}} = 87.13 \text{ Min.}} \quad \textbf{Ans.}$$

Example 7. A satellite is moving in an elliptical transfer orbit with apogee and perigee at a distance of 35000 Km and 500 Km. If the radius of earth is 6360 Km. Determine the velocity of a satellite at any point in its orbit.

Solution. The velocity (V) at any point along an elliptical orbit is given by

$$V = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

Given that,

$$\mu = GM = 6.67 \times 10^{-11} \times 5.98 \times 10^{24}$$
$$= 39.8 \times 10^{13} \text{ Nm}^2\text{.kg}^2$$
$$a = 35000 \text{ Km}$$
$$r = \text{Apogee distance from the center of earth}$$
$$r = 35000 + 6360 = 41360 \text{ Km}$$

Putting all these values, we get

$$V = \sqrt{39.8 \times 10^{13}\left(\frac{2}{41360 \times 10^3} - \frac{1}{35000 \times 10^3}\right)}$$

$$= \sqrt{39.8 \times 10^{10}\left(\frac{2}{41360} - \frac{1}{35000}\right)}$$

$$= \sqrt{39.8 \times 10^4 \,(48.3559 - 28.5714)}$$

$$= \sqrt{39.8 \times 10^4 \times 19.7845}$$

$$= 10^2 \times 28.061 = 2.81 \text{ Km/sec.}$$

$$\boxed{V = 2.81 \text{ Km/sec}} \quad \textbf{Ans.}$$

Example 8. Determine the orbit eccentrcity of a satellite moving in an elliptical eccentric orbit having the semi-major axis equal to 16000 Km. If the difference between the apogee and perigee is 25000 Km, and the earth's radius is 6360 Km.

Solution. Apogee $= a(1 + e)$

Perigee $= a(1 - e)$

where a = Semi-major axis = 16000 Km

e = Orbital eccentricity

Given that,

$$\begin{aligned} \text{Apogee} - \text{Perigee} &= 25000 \text{ Km} \\ a(1+e) - a(1-e) &= 25000 \text{ Km} \\ 2ae &= 25000 \text{ Km} \\ e \times 2 \times 16000 &= 25000 \\ e &= \frac{25000}{32000} = \frac{25}{32} \end{aligned}$$

$$\boxed{e = 0.78} \quad \textbf{Ans.}$$

Example 9. Two satellites are moving in an elliptical eccentric orbit with same perigee but different apogee distances. Satellite 1 is having an orbital period of 5 hours and semimajor axis, 20000 Km. While the orbital period of satellite 2 is 2 hours 50 min. Determine the semimajor axis of satellite.

Solution. Orbital period is given by

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

where $\mu = GM = 3.98 \times 10^{13}$ Nm^2/kg

if T_1 and T_2 are the orbital period of satellite-1 and satellite-2 respectively, then,

$$\frac{T_1}{T_2} = \frac{2\pi\sqrt{\frac{a_1^3}{\mu}}}{2\pi\sqrt{\frac{a_2^3}{\mu}}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{a_1^3}{a_2^3}}$$

Given that, T_1 = 5 hours = 300 min

T_2 = 2 hours, 50 min = 170 min

a_1 = 20000 Km

Substituting these values in above formula, we get

$$\frac{300}{170} = \sqrt{\frac{(20000)^3}{a_2^3}}$$

$$\Rightarrow \quad \frac{(20000)^3}{a_2^3} = 3.114$$

$$\Rightarrow \quad \frac{20000}{a_2} = 1.460 \Rightarrow a_2 = 13698.63 \text{ Km}$$

$$\boxed{a_2 = 13698.63 \text{ Km}} \quad \textbf{Ans.}$$